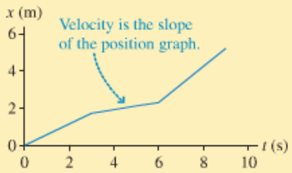
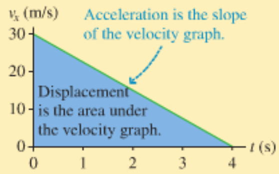
Kinematics Review

A **position-versus-time graph** plots position on the vertical axis against time on the horizontal axis.

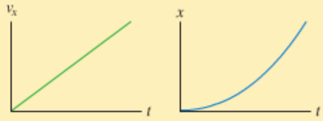


A **velocity-versus-time graph** plots velocity on the vertical axis against time on the horizontal axis.



Motion with constant acceleration

An object with constant acceleration has a constantly changing velocity. Its velocity graph is linear; its position graph is a parabola.



Kinematic equations for motion with constant acceleration:

$$(v_x)_f = (v_x)_i + a_x \Delta t$$

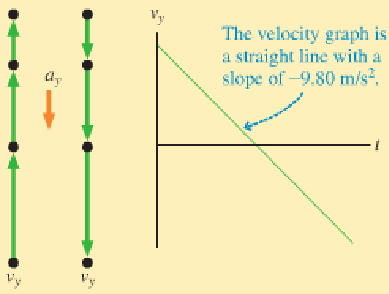
$$x_f = x_i + (v_x)_i \Delta t + \frac{1}{2}a_x(\Delta t)^2$$

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \Delta x$$

Free Fall

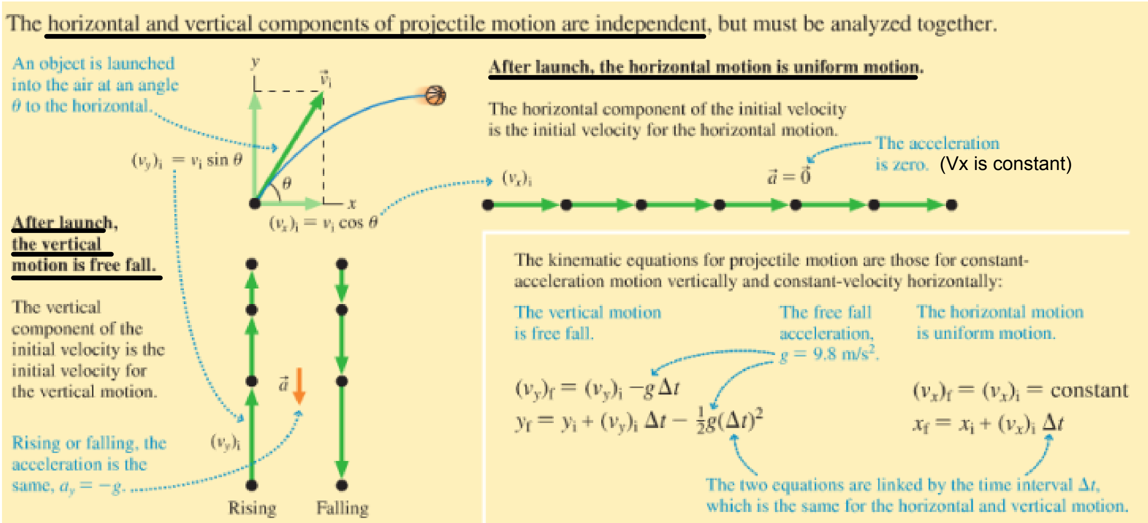
Free fall

Free fall is a special case of constant-acceleration motion. The acceleration has magnitude $g = 9.80\ \text{m/s}^2$ and is always directed vertically downward whether an object is moving up or down.

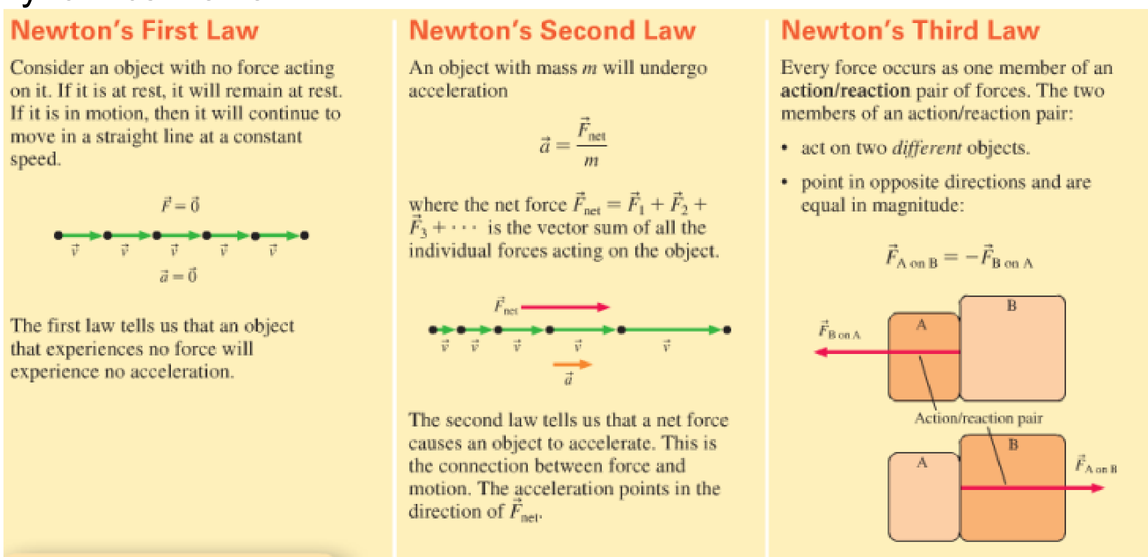


*Free fall means the only force acting on the object is the force of gravity (no air resistance!)

Projectile Motion



Dynamics Review



Equilibrium - Fnet = 0 in either of these situations! (Fnet,x = 0, Fnet,y = 0)

- static - at rest
- dynamic - moving in a straight line at constant speed

Object is accelerating? Then...

$$\sum F_x = ma_x \quad \text{and} \quad \sum F_y = ma_y$$

Newton's second law in component form

Let's define your **apparent weight** w_{app} in terms of the force you feel:

$$w_{app} = \text{magnitude of supporting contact forces}$$

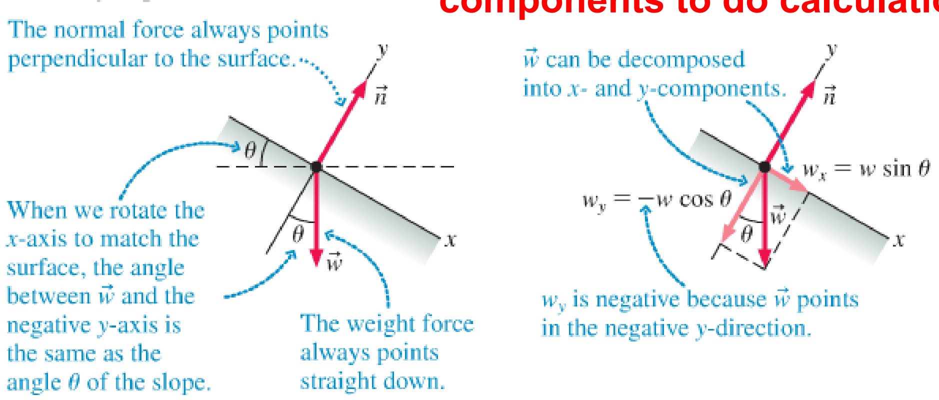
Definition of apparent weight

A person in free fal has zero apparent weight!

Inclined Planes

AP Test: Must draw diagram with original forces only first, then may draw again with components to do calculations!

(a) Analyzing forces on an incline

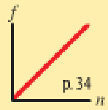


Friction

$$\text{Static: } \vec{f}_s = (\text{magnitude} \le f_{s\,max} = \mu_s n, \text{ direction as necessary to prevent motion})$$

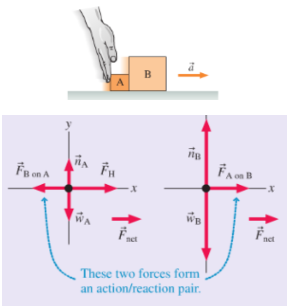
$$\text{Kinetic: } \vec{f}_k = (\mu_k n, \text{ direction opposite the motion}) \qquad (5.10)$$

$$\text{Rolling: } \vec{f}_r = (\mu_r n, \text{ direction opposite the motion})$$



Interacting Objects

FIGURE 5.25 Two boxes moving together have the same acceleration.



TACTICS BOX 5.2 Working with objects in contact

When two objects are in contact and their motion is linked, we need to duplicate certain steps in our analysis:

1. Draw each object separately and prepare a separate force identification diagram for *each* object.
2. Draw a separate free-body diagram for *each* object.
3. Write Newton's second law in component form for *each* object.

The two objects in contact exert forces on each other:

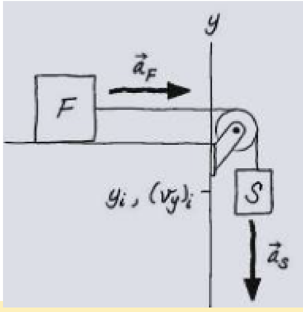
4. Identify the action/reaction pairs of forces. If object A acts on object B with force $\vec{F}_{A\ on\ B}$, then identify the force $\vec{F}_{B\ on\ A}$ that B exerts on A.
5. Newton's third law says that you can equate the magnitudes of the two forces in each action/reaction pair.

The fact that the objects are in contact simplifies the kinematics:

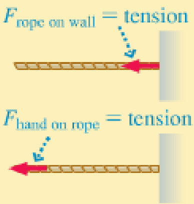
6. Objects in contact will have the same acceleration.

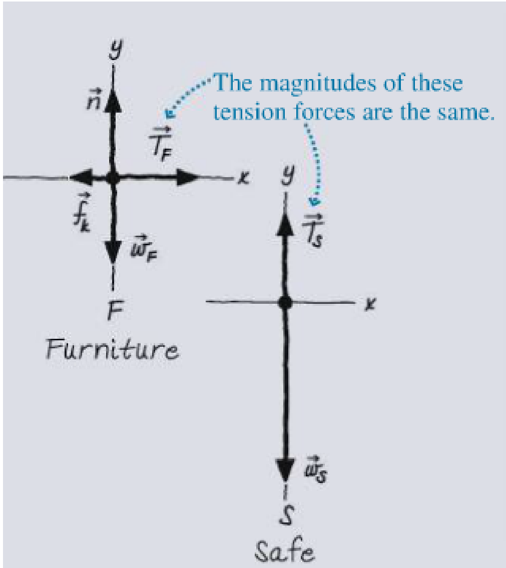
Exercises 25, 26, 27

Strings and Pulleys



- A string or rope pulls what it's connected to with a force equal to its tension.
- The tension in a rope is equal to the force pulling on the rope.
- The tension in a massless rope is the same at all points in the rope.
- Tension does not change when a rope passes over a massless, frictionless pulley.





Uniform Circular Motion

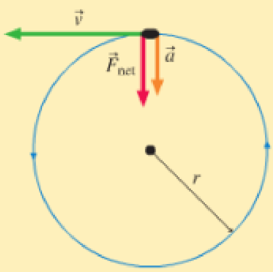
An object moving in a circular path is in uniform circular motion if v is constant.

- The speed is constant, but the direction of motion is constantly changing.
- The **centripetal acceleration** is directed toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

- This acceleration requires a net force directed toward the center of the circle. Newton's second law for circular motion is

$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$



Describing circular motion

For an object moving in a circle of radius r at a constant speed v:

- The **period** T is the time to go once around the circle.

$$T = \text{time for one revolution}$$

- The **frequency** f is defined as the number of revolutions per second. It is defined in terms of the period:

$$f = \frac{1}{T}$$

- The frequency and period are related to the speed and the radius:

$$v = 2\pi f r = \frac{2\pi r}{T}$$

Universal Gravitation

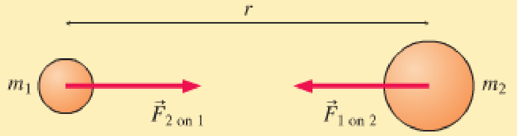
Two objects with masses m_1 and m_2 that are distance r apart exert attractive gravitational forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

where the gravitational constant is

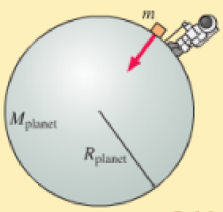
$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

This is **Newton's law of gravity.** Gravity is an inverse-square law.



Planetary gravity

The gravitational attraction between a planet and a mass on the surface depends on the two masses and the distance to the center of the planet.



$$F_{\text{planet on } m} = \frac{GM_{\text{planet}}m}{R_{\text{planet}}^2}$$

We can use this to define a value of the free-fall acceleration at the surface of a planet:

$$g_{\text{planet}} = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2}$$

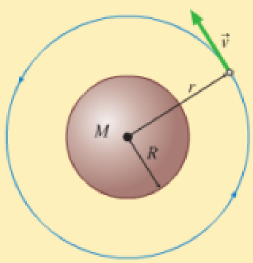
Orbital motion

A **satellite** in a circular orbit of radius r around an object of mass M moves at a speed v given by

$$v = \sqrt{\frac{GM}{r}}$$

The period and radius are related as follows:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$



The speed of a satellite in a low orbit is

$$v = \sqrt{gr}$$

The orbital period is

$$T = 2\pi\sqrt{\frac{r}{g}}$$

Rotational Motion

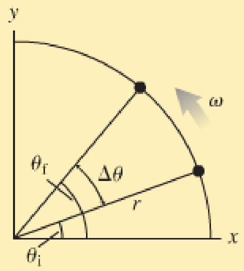
Describing circular motion

We define new variables for circular motion. By convention, counterclockwise is positive.

Angular displacement: $\Delta\theta = \theta_f - \theta_i$

Angular velocity: $\omega = \dfrac{\Delta\theta}{\Delta t}$

Angular acceleration: $\alpha = \dfrac{\Delta\omega}{\Delta t}$



Angles are measured in radians:

1 rev = 360° = 2π rad

The angular velocity depends on the frequency and period: $\omega = \dfrac{2\pi}{T} = 2\pi f$

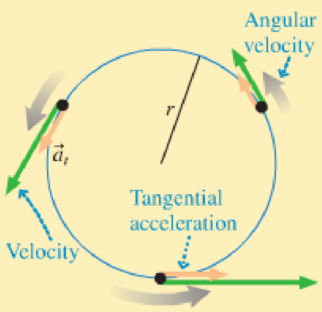
ω is the same for all points on a carousel because all do the same angle in the same time! The linear velocity "v" is different depending on the radius! (Linear stuff depends on radius!)

Relating linear and circular motion quantities

Linear and angular speeds are related by: $v = \omega r$

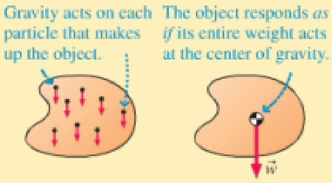
If the particle's speed is increasing, it will also have a tangential acceleration $\vec{a}_t$ directed tangent to the circle and an angular acceleration α.

Angular and tangential accelerations are related by: $a_t = \alpha r$



Center of gravity

The **center of gravity** of an object is the point at which gravity can be considered to be acting.



The **position of the center of gravity** depends on the distance $x_1, x_2, \ldots$ of each particle of mass $m_1, m_2, \ldots$ from the origin:

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

The **moment of inertia** is the rotational equivalent of mass. For an object made up of particles of masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis, the moment of inertia is

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = mr^2$$

Newton's Second Law for Rotational Motion

If a net torque τ_{net} acts on an object, the object will experience an angular acceleration given by $\alpha = \tau_{net}/I$, where I is the object's moment of inertia about the rotation axis.

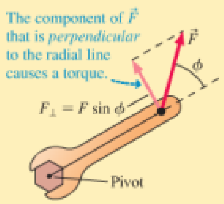
This law is analogous to Newton's second law for linear motion, $\vec{a} = \vec{F}_{net}/m$.

Torque

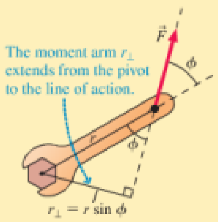
A force causes an object to undergo a linear acceleration, a torque causes an object to undergo an angular acceleration.

There are two interpretations of torque:

Interpretation 1: $\tau = rF_\perp$



Interpretation 2: $\tau = r_\perp F$



Both interpretations give the same expression for the magnitude of the torque: $\tau = rF \sin\phi$

Some special cases for rotational motion:

If a rope unwinds from a pulley of radius R, the linear motion of an object tied to the rope is related to the angular motion of the pulley by

$$a_{\text{obj}} = \alpha R \qquad v_{\text{obj}} = \omega R$$

For an object that rolls without slipping,

$$v = \omega R$$

(If you are doing an experiment problem, you might want to say that you are making the assumption that it is not slipping.)

Static Equilibrium for Extended Objects

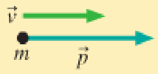
$$\left.\begin{aligned}\sum F_x &= 0\\ \sum F_y &= 0\end{aligned}\right\}\ \text{No net force}$$

$$\sum \tau = 0 \ \}\ \text{No net torque}$$

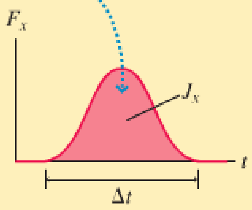
The net torque about every point is zero so any point can be the pivot. (Pick the one where you have the most complicated forces so all of those forces can be ignored because a force through the axis of rotation causes no torque.)

Impulse and Momentum

Momentum $\vec{p} = m\vec{v}$



Impulse J_x = area under force curve



Impulse and momentum are related by the **impulse-momentum theorem**

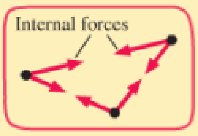
$$\Delta p_x = J_x$$

This is an alternative statement of Newton's second law.

Angular momentum $L = I\omega$ is the rotational analog of linear momentum $\vec{p} = m\vec{v}$.

System A group of interacting particles

Isolated system A system on which the net external force is zero



Conservation of momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an **isolated system**—one on which no net force acts—is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Conservation of angular momentum

The angular momentum L of a rotating object or system of objects subject to zero net external torque is a constant. Thus

$$L_f = L_i$$

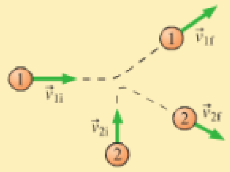
This can be written in terms of the initial and final moments of inertia I and angular velocities ω as

$$(I_1)_f(\omega_1)_f + (I_2)_f(\omega_2)_f + \cdots = (I_1)_i(\omega_1)_i + (I_2)_i(\omega_2)_i + \cdots$$

Collisions Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.

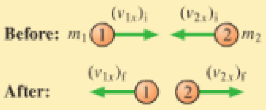
Explosions Two or more particles move away from each other.

Two dimensions Both the x- and y-components of the total momentum $\vec{P}$ must be conserved, giving two simultaneous equations.



Before-and-after visual overview

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.



$$m_1(v_{1x})_f + m_2(v_{2x})_f + \cdots = m_1(v_{1x})_i + m_2(v_{2x})_i + \cdots$$
$$m_1(v_{1y})_f + m_2(v_{2y})_f + \cdots = m_1(v_{1y})_i + m_2(v_{2y})_i + \cdots$$

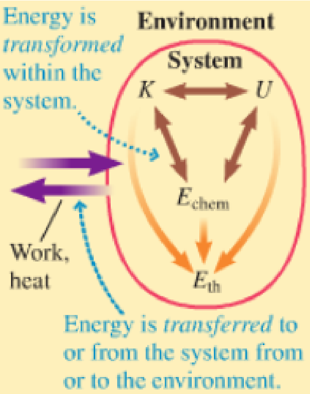
Conservation of Energy

Basic Energy Model

Within a system, energy can be **transformed** between various forms.

Energy can be **transferred** into or out of a system in two basic ways:

- **Work:** The transfer of energy by mechanical forces
- **Heat:** The nonmechanical transfer of energy from a hotter to a colder object



Conservation of Energy

When work W is done on a system, the system's total energy changes by the amount of work done. In mathematical form, this is the **work-energy equation:**

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = W$$

A system is **isolated** when no energy is transferred into or out of the system. This means the work is zero, giving the **law of conservation of energy:**

$$\Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = 0$$

Solving Energy Transfer and Energy Conservation Problems

PREPARE Draw a before-and-after visual overview.

SOLVE

- If work is done on the system, then use the before-and-after version of the work-energy equation:

$$K_f + (U_g)_f + (U_s)_f + \Delta E_{th} = K_i + (U_g)_i + (U_s)_i + W$$

Energy into/out of system

- If the system is isolated but there's friction present, then the total energy is conserved:

$$K_f + (U_g)_f + (U_s)_f + \Delta E_{th} = K_i + (U_g)_i + (U_s)_i$$

- If the system is isolated and there's no friction, (isolated means no work!) then mechanical energy is conserved:

$$K_f + (U_g)_f + (U_s)_f = K_i + (U_g)_i + (U_s)_i$$

ASSESS Kinetic energy is always positive, as is the change in thermal energy.

Kinetic energy is an energy of motion:

$$K = \underbrace{\frac{1}{2}mv^2}_{\text{Translational}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{Rotational}}$$

Potential energy is energy stored in a system of interacting objects.

- **Gravitational potential energy:** $U_g = mgy$
- **Elastic potential energy:** $U_s = \frac{1}{2}kx^2$

Mechanical energy is the sum of a system's kinetic and potential energies:

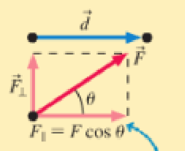
$$\text{Mechanical energy} = K + U = K + U_g + U_s$$

Thermal energy is the sum of the microscopic kinetic and potential energies of all the molecules in an object. The hotter an object, the more thermal energy it has. When kinetic (sliding) friction is present, the increase in the thermal energy is $\Delta E_{th} = f_k \Delta x$.

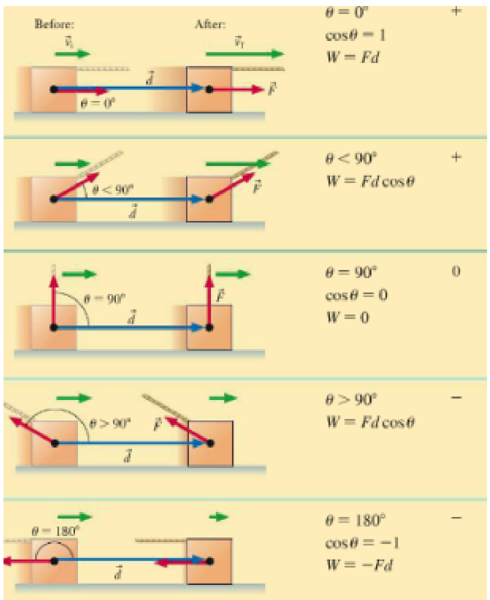
Work is the process by which energy is transferred to or from a system by the application of mechanical forces.

If a particle moves through a displacement $\vec{d}$ while acted upon by a constant force $\vec{F}$, the force does work

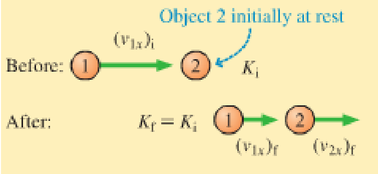
$$W = F_{\parallel}d = Fd\cos\theta$$



★ Only the component of the force parallel to the displacement does work.



Perfectly elastic collisions
Both mechanical energy and momentum are conserved.



Power is the rate at which energy is transformed . . .

$$P = \frac{\Delta E}{\Delta t}$$

ΔE ← Amount of energy transformed
Δt ← Time required to transform it

. . . or at which work is done.

$$P = \frac{W}{\Delta t}$$

W ← Amount of work done
Δt ← Time required to do work

*You may see the AP test use the definition instead of the term! (Like find the rate at which energy is converted from ... to ..., which means find power!)

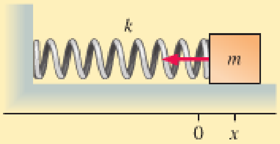
Oslillations

force on spring increases linearly as you pull back (restoring means it is opposite the direction you pull) - Hooke's Law

Frequency and Period

SHM occurs when a <u>linear restoring force</u> acts to return a system to an equilibrium position. Frequency and the period depend on the force and on masses or lengths. <u>Frequency and period do not depend on amplitude</u>.

Mass on spring

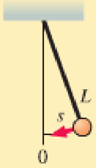


$$(F_{net})_x = -kx$$

The frequency and period of a mass on a spring depend on the mass and the spring constant: They are the same for horizontal and vertical systems.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Pendulum



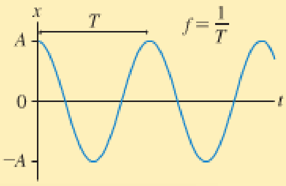
$$(F_{net})_t = -\left(\frac{mg}{L}\right)s$$

The frequency and period of a pendulum depend on the length and the free-fall acceleration. They do not depend on the mass.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

Oscillation

An **oscillation** is a repetitive motion about an equilibrium position. The **amplitude** A is the maximum displacement from equilibrium. The period T is the time for one cycle. We may also characterize an oscillation by its frequency f.

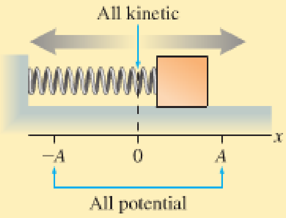


$$f = \frac{1}{T}$$

Energy

If there is no friction or dissipation, kinetic and potential energies are alternately transformed into each other in SHM, with the sum of the two conserved.

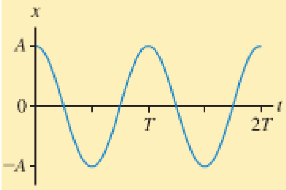
$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$$



Simple Harmonic Motion (SHM)

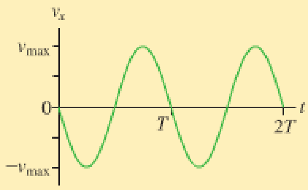
SHM is an oscillation that is described by a sinusoidal function. All systems that undergo SHM can be described by the same functional forms.

Position-versus-time is a cosine function.



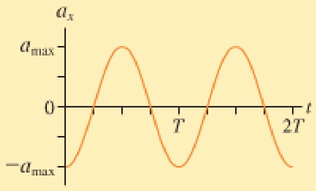
$x(t) = A\cos(2\pi ft)$

Velocity-versus-time is an inverted sine function.



$v_x(t) = -[(2\pi f)A]\sin(2\pi ft)$

Acceleration-versus-time is an inverted cosine function.

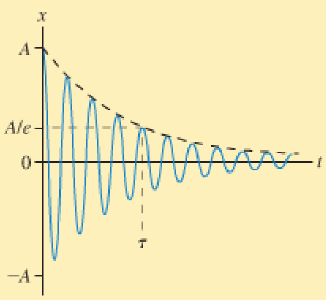


$a_x(t) = -[(2\pi f)^2 A]\cos(2\pi ft)$

reflections of each other

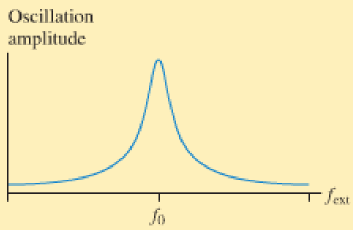
Damping

Simple harmonic motion with damping (due to drag) decreases in amplitude over time. The **time constant** τ determines how quickly the amplitude decays.



Resonance

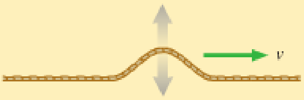
A system that oscillates has a **natural frequency** of oscillation f_0. **Resonance** occurs if the system is driven with a frequency f_{ext} that matches this natural frequency. This may produce a large amplitude of oscillation.



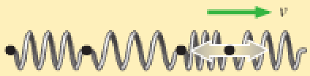
Waves

This model is based on the idea of a **traveling wave,** which is an organized disturbance traveling at a well-defined **wave speed** v.

- In **transverse waves** the particles of the medium move *perpendicular* to the direction in which the wave travels.



- In **longitudinal waves** the particles of the medium move *parallel* to the direction in which the wave travels.



A wave transfers energy, but there is no material or substance transferred.

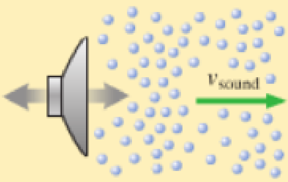
Mechanical waves require a material **medium.** The speed of the wave is a property of the medium, not the wave. The speed does not depend on the size or shape of the wave.

- For a **wave on a string,** the string is the medium.

$$v_{string} = \sqrt{\frac{T_s}{\mu}}$$

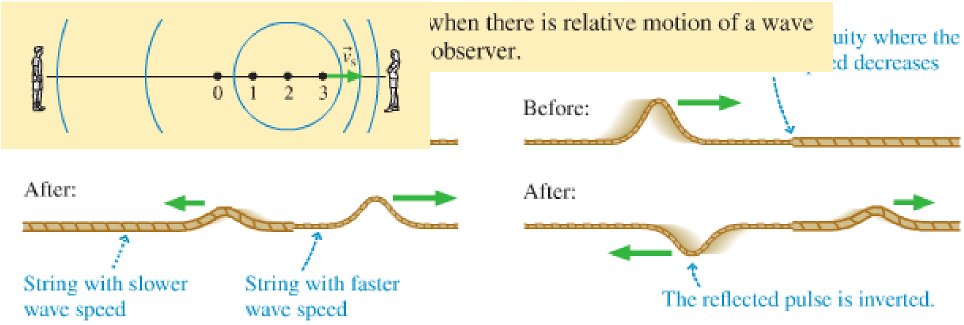
Don't worry about equation, but realize velocity depends on tension and density and string (so the medium)

- A **sound wave** is a wave of compressions and rarefactions of a medium such as air.



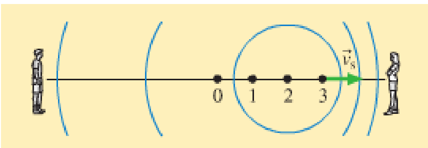
In a gas:

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}}$$



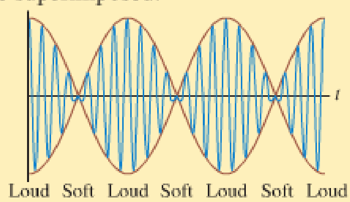
*Speed changes when medium changes
*frequency remains constant
*since speed changes, wavelength changes
*amplitude related to energy to reflected is always less amplitude

The **Doppler effect** is a shift in frequency when there is relative motion of a wave source (frequency f_s, wave speed v) and an observer.



closer - higher pitch

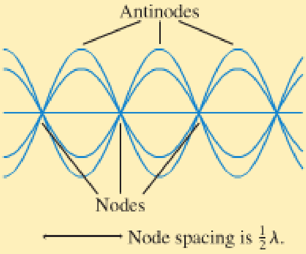
Beats (loud-soft-loud-soft modulations of intensity) are produced when two waves of slightly different frequencies are superimposed.



$$f_{beat} = |f_1 - f_2|$$

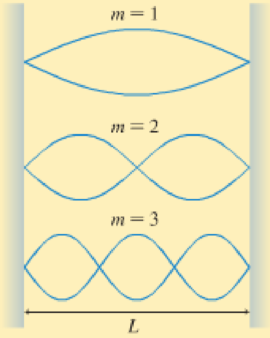
Standing Waves

Two identical traveling waves moving in opposite directions create a standing wave.



The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are **modes** of the system.

A **standing wave on a string** has a node at each end. Possible modes:



$$\lambda_m = \frac{2L}{m} \qquad f_m = m\left(\frac{v}{2L}\right) = mf_1$$

$$m = 1, 2, 3, \ldots$$

A **standing sound wave in a tube** can have different boundary conditions: open-open, closed-closed, or open-closed.

Open-open

$$f_m = m\left(\frac{v}{2L}\right)$$

$m = 1, 2, 3, \ldots$

same as string

Closed-closed

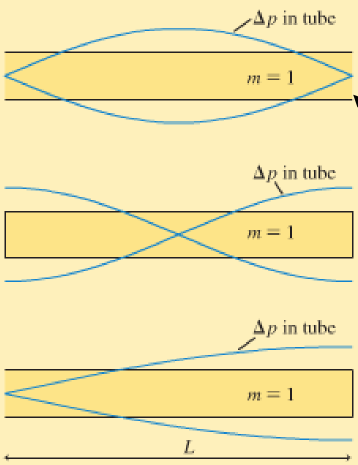
$$f_m = m\left(\frac{v}{2L}\right)$$

$m = 1, 2, 3, \ldots$

Open-closed

$$f_m = m\left(\frac{v}{4L}\right)$$

$m = 1, 3, 5, \ldots$



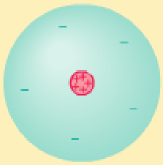
(Open end = pressure node or displacement antinode)

open-open or closed-closed get 1/2 a wavelength to fit inside at fundamental frequency

open-closed only get 1/4 of wavelength to fit

Electrostatics

There are two kinds of charges, called **positive** and **negative.**



- Atoms consist of a nucleus containing positively charged protons surrounded by a cloud of negatively charged electrons.
- The **fundamental charge** e is the magnitude of the charge on an electron or proton: $e = 1.60 \times 10^{-19}$ C.
- Matter with equal amounts of positive and negative charge is **neutral.**
- Charge is conserved; it can't be created or destroyed.

Coulomb's Law

The forces between two charged particles q_1 and q_2 separated by distance r are

$$F_{1\,\text{on}\,2} = F_{2\,\text{on}\,1} = \frac{K|q_1||q_2|}{r^2}$$

where $K = 8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$ is the **electrostatic constant.** These forces are an action/reaction pair directed along the line joining the particles.

- The forces are repulsive for two like charges, attractive for two opposite charges.
- The net force on a charge is the vector sum of the forces from all other charges.
- The unit of charge is the coulomb (C).

There are two types of material, **insulators** and **conductors.**

- Charge remains fixed on an insulator.
- Charge moves easily through conductors.
- Charge is transferred by contact between objects.

Circuits

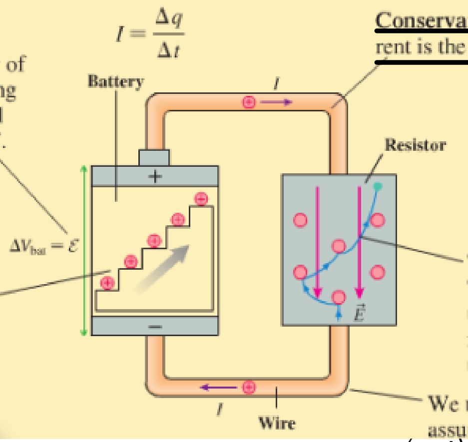
Batteries, Resistors, and Current

The **current** is defined to be the motion of positive charges

$$I = \frac{\Delta q}{\Delta t}$$

The battery transforms its chemical energy into electric potential energy of the charges passing through it, raising their electric potential. The potential difference of the battery is its **emf** $\mathcal{E}$.

A **battery** is a source of potential difference. Chemical processes in the battery separate charges. We use a **charge escalator** model to show the lifting of charges to higher potential.



Conservation of current dictates that the current is the same at all points in the circuit.

The battery creates an electric field in the circuit that causes charges to move. Positive charges move in the direction of the electric field, which is the direction of decreasing potential.

The actual charge carriers are electrons. Their random collisions with atoms impede the flow of charge and are the source of **resistance.** The collisions increase the thermal energy of the **resistor.**

We use the **ideal-wire model** in which we assume that there is no resistance in the wires.
(so $\Delta V = 0$ for wire - no energy to get through wire)

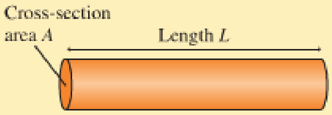
Resistance, resistivity, and Ohm's law

The **resistivity** ρ is a property of a material, a measure of how good a conductor the material is.

- Good conductors have low resistivity.
- Poor conductors have high resistivity.

The **resistance** is a property of a particular wire or conductor. The resistance of a wire depends on its resistivity and dimensions.

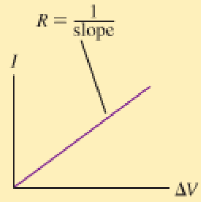
$$R = \frac{\rho L}{A}$$



Ohm's law describes the relationship between potential difference and current in a resistor:

$$I = \frac{\Delta V}{R}$$

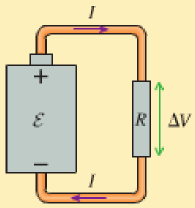
The resistance is



If a resistor is ohmic, then the resistance is constant. So it you double potential, you would double the current.

Energy and power

The energy used by a circuit is supplied by the emf of the battery through a series of energy transformations:



$$E_{chem} \rightarrow U_{elec} \rightarrow K \rightarrow E_{th}$$

Chemical energy in the battery

Potential energy of separated charges

Kinetic energy of moving charges

Thermal energy of atoms in the resistor

Power is always the rate that energy is transformed.

The battery *supplies* power at the rate

$$P_{emf} = I\mathcal{E}$$

The resistor *dissipates* power at the rate

$$P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R}$$

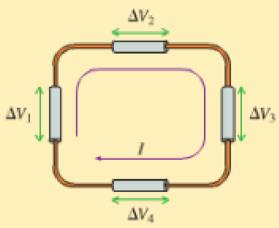
Be sure to know all forms of power equation and when to use which one! (If something is constant, and you want to compare power, pick that one!)

Kirchhoff's loop law

For a closed loop:

- Assign a direction to the current.
- Add potential differences around the loop:

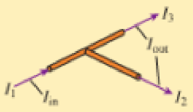
$$\sum_i \Delta V_i = 0$$



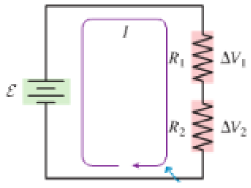
Kirchhoff's junction law

For a junction:

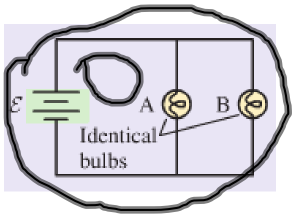
$$\sum I_{in} = \sum I_{out}$$



(a) Two resistors in series



$\Delta V_1 + \Delta V_2 = \Delta V_{battery}$

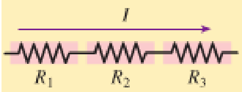


$\Delta V_A = \Delta V_{battery}$ $\Delta V_B = \Delta V_{battery}$

Series elements

A series connection has no junction.
The current in each element is the same.

Resistors in series can be reduced to an equivalent resistance:



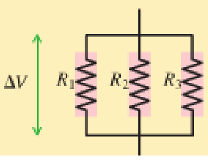
$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

R_{eq} higher than any individual resistor.

Parallel elements

Elements connected in parallel are connected by wires at both ends.
The potential difference across each element is the same.

Resistors in parallel can be reduced to an equivalent resistance:



$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1}$$

R_{eq} lower than any individual resistor.